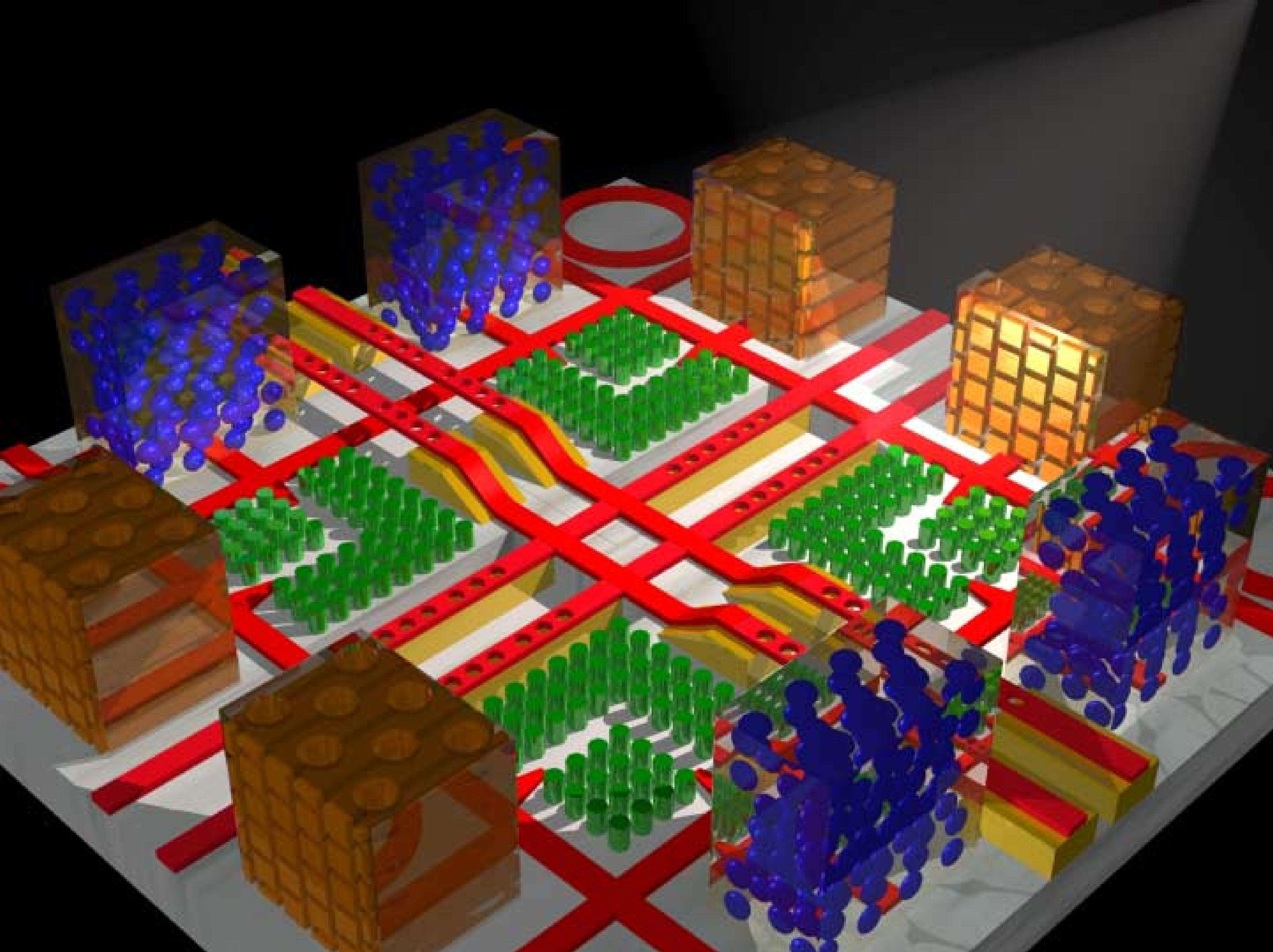
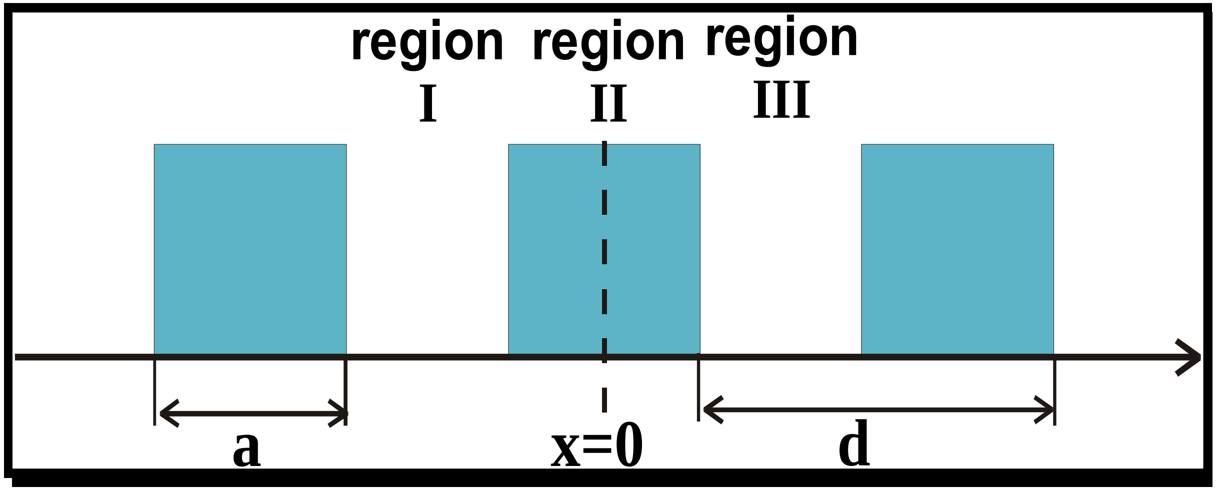


1D multilayer structure



$\varepsilon(x) = \varepsilon$ for slabs,
$\varepsilon(x) = 1$ elsewhere

Maxwell's equations in the 1D case are reduced to $$\frac{\partial^2 E(x,t)}{\partial x^2} = \frac{\varepsilon(x)}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2}$$

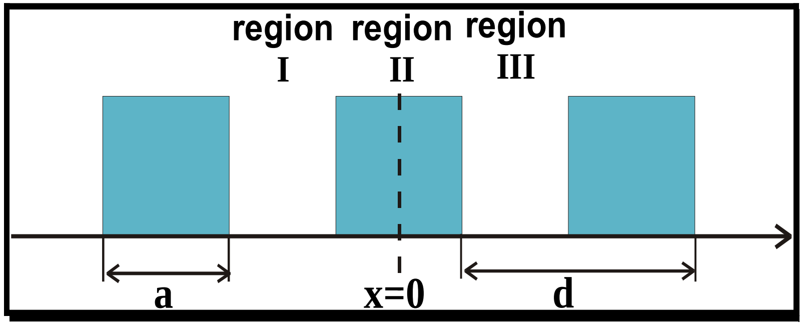
$$E(x,t) = E(x)\exp(-i\omega t) \longrightarrow \frac{\partial^2 E(x)}{\partial x^2} + \frac{\varepsilon(x)\omega^2}{c^2}E(x) = 0$$

Mathematically, the 1-D wave equation is similar to the Schroedinger equation. By analogy, positive dielectric scatterers are similar to regions of negative potential energy in a quantum system

Translation invariance and boundary conditions

Since $\varepsilon(x)$ remains unchanged when the structure is translated by d

$$E(x+d) = \exp(igd)\,E(x)$$



where g is the crystal momentum, or the Bloch vector

Plane-wave solutions for regions I – III

Region I: $E_I(x) = A\exp(ikx) + B\exp(-ikx)$

Region II: $E_{II}(x) = f\exp(iqx) + g\exp(-iqx)$

Region III: $E_{III}(x) = C\exp(ikx) + D\exp(-ikx)$

$$k = \omega/c$$

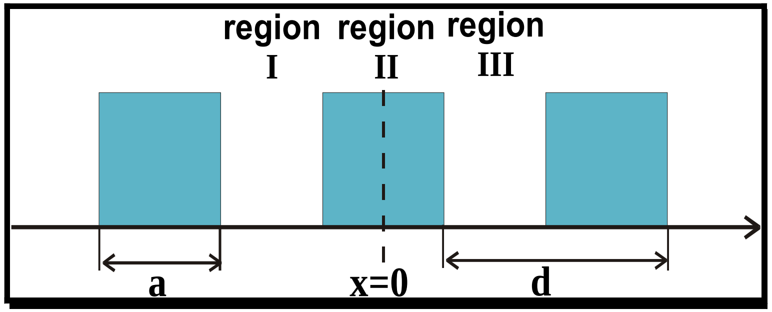
$$q = \sqrt{\varepsilon}\,\omega/c$$

Boundary conditions

$$E_I(x) = E_{II}(x)\Big|_{x=-a/2} \qquad E'_I(x) = E'_{II}(x)\Big|_{x=-a/2}$$

$$E_{II}(x) = E_{III}(x)\Big|_{x=+a/2} \qquad E'_{II}(x) = E'_{III}(x)\Big|_{x=+a/2}$$

Dispersion relation for an infinite periodic multilayer



Matrix relation for plane-wave amplitudes

$$\begin{pmatrix} C \\ D \end{pmatrix} = M \begin{pmatrix} A \\ B \end{pmatrix}$$

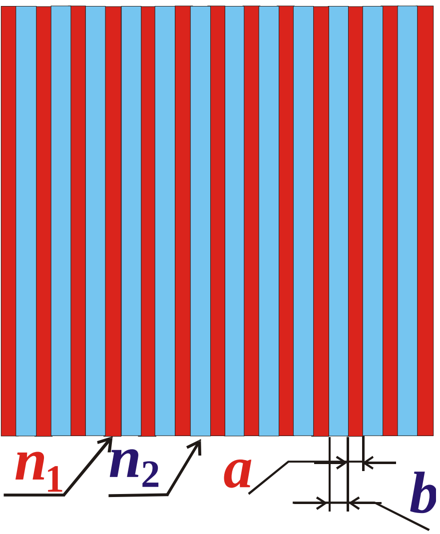
$$M = \begin{pmatrix} \exp(-ika)\left[\cos(qa) + i\left(\frac{k}{q} + \frac{q}{k}\right)\frac{\sin(qa)}{2}\right] & i\left(\frac{q}{k} - \frac{k}{q}\right)\frac{\sin(qa)}{2} \\ -i\left(\frac{q}{k} - \frac{k}{q}\right)\frac{\sin(qa)}{2} & \exp(ika)\left[\cos(qa) - i\left(\frac{k}{q} + \frac{q}{k}\right)\frac{\sin(qa)}{2}\right] \end{pmatrix}$$

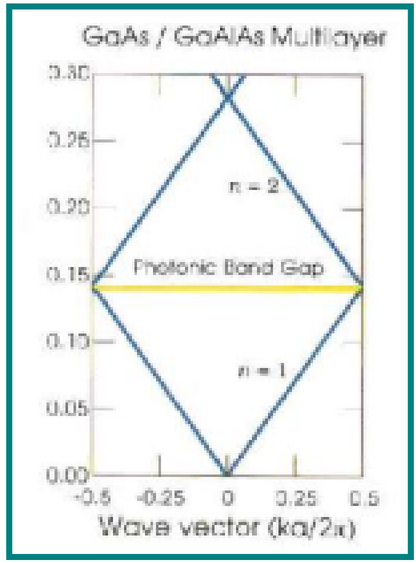
Characteristic equation: $TM\begin{pmatrix} A \\ B \end{pmatrix} = \exp(igd)\begin{pmatrix} A \\ B \end{pmatrix}$ $\quad T = \begin{pmatrix} \exp(ikd) & 0 \\ 0 & \exp(-ikd) \end{pmatrix}$

The dispersion relation is then given by

$$\cos(gd) = \cos\left[k(d-a)\right]\cos(qa) - \frac{1}{2}\left(\frac{k}{q} + \frac{q}{k}\right)\sin\left[k(d-a)\right]\sin(qa)$$

Dispersion of a periodic multilayer





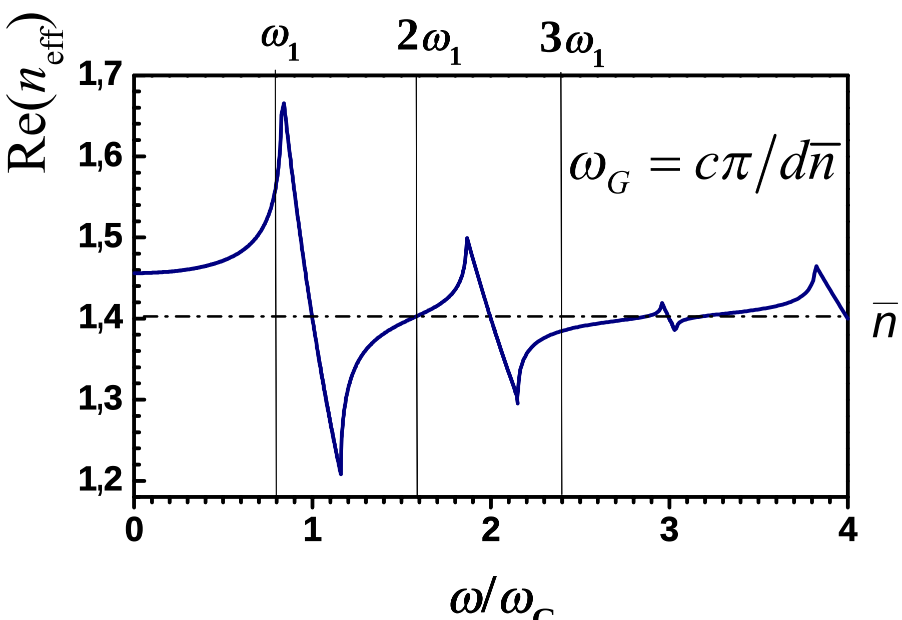
Bloch wave

$$E(x, z, t) = E_K(x) \exp\left[i \left(Kx + \beta z - \omega t \right) \right]$$

Dispersion relation

$$\cos(K\Lambda) = \cos(k_{1x}a)\cos(k_{2x}b) - \Delta \sin(k_{1x}a)\sin(k_{2x}b)$$

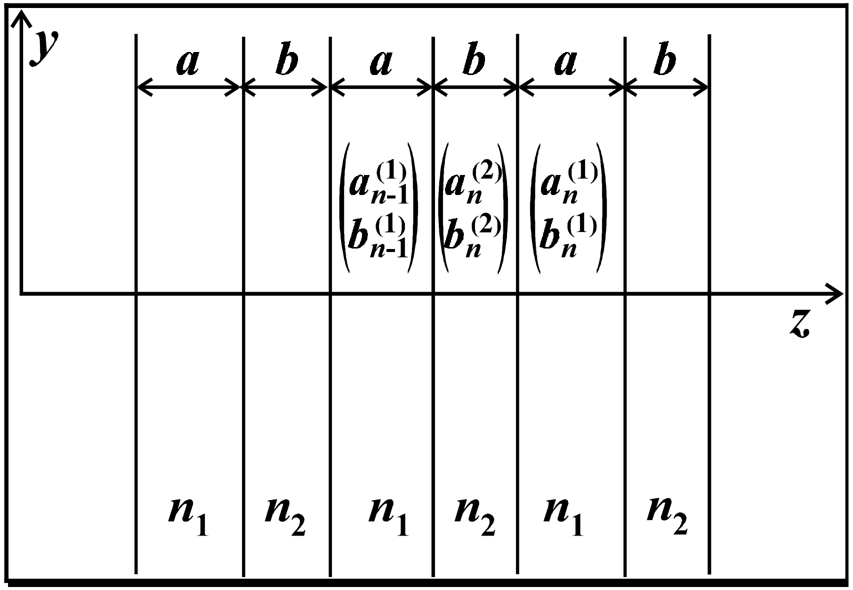
$$\Delta = \begin{cases} \dfrac{1}{2}\left(\dfrac{k_{1x}}{k_{2x}} + \dfrac{k_{2x}}{k_{1x}} \right), & \text{TE waves} \\[2ex] \dfrac{1}{2}\left(\dfrac{n_2^2 k_{1x}}{n_1^2 k_{2x}} + \dfrac{n_1^2 k_{2x}}{n_2^2 k_{1x}} \right), & \text{TM waves} \end{cases}$$



$n_{eff} = Kc/\omega$, $a/d = b/d = 0.5$,
$n_1 = 1.8$, $n_2 = 1$

Finite 1D PBG structure

$$E(y,z) = \left\{ a_n^{(\alpha)} \exp\left[-ik_{\alpha z}(z - nd) \right] + b_n^{(\alpha)} \exp\left[ik_{\alpha z}(z - nd) \right] \right\} \exp(-ik_y y)$$



$$k_{\alpha z} = \sqrt{\left(\frac{n_\alpha \omega}{c} \right)^2 - k_y^2} \qquad \alpha = 1, 2$$

In the case of TE-waves

(**E** is perpendicular to the *yz*-plane)

$$a_{n-1} + b_{n-1} = c_n \exp(ik_{2z} d) + d_n \exp(-ik_{2z} d)$$

$$ik_{1z}(a_{n-1} - b_{n-1}) = ik_{2z}\left(c_n \exp(ik_{2z} d) - d_n \exp(-ik_{2z} d) \right)$$

$$c_n \exp(ik_{2z} a) + d_n \exp(-ik_{2z} a) = a_n \exp(ik_{1z} a) + b_n \exp(-ik_{1z} a)$$

$$ik_{2z}\left(c_n \exp(ik_{2z} a) - d_n \exp(-ik_{2z} a) \right) = ik_{1z}\left(a_n \exp(ik_{1z} d) - b_n \exp(-ik_{1z} d) \right)$$

Finite 1D PBG structure

$$E(y,z) = \left\{a_n^{(\alpha)} \exp\left[-ik_{\alpha z}(z-nd)\right] + b_n^{(\alpha)} \exp\left[ik_{\alpha z}(z-nd)\right]\right\} \exp(-ik_y y)$$

$$k_{\alpha z} = \sqrt{\left(\frac{n_\alpha \omega}{c}\right)^2 - k_y^2} \qquad \alpha = 1, 2$$

$$a_{n-1} + b_{n-1} = c_n \exp(ik_{2z} d) + d_n \exp(-ik_{2z} d)$$

$$ik_{1z}(a_{n-1} - b_{n-1}) = ik_{2z}\left(c_n \exp(ik_{2z} d) - d_n \exp(-ik_{2z} d)\right)$$

$$c_n \exp(ik_{2z} a) + d_n \exp(-ik_{2z} a) = a_n \exp(ik_{1z} a) + b_n \exp(-ik_{1z} a)$$

$$ik_{2z}\left(c_n \exp(ik_{2z} a) - d_n \exp(-ik_{2z} a)\right) = ik_{1z}\left(a_n \exp(ik_{1z} d) - b_n \exp(-ik_{1z} d)\right)$$

Transfer matrix and reflection/transmission spectra

Relation between plane-wave amplitudes

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

$$A = \exp\left(ik_{1z}a\right)\left[\cos\left(k_{2z}b\right) + \frac{i}{2}\left(\frac{k_{2z}}{k_{1z}} + \frac{k_{1z}}{k_{2z}}\right)\sin\left(k_{2z}b\right)\right]$$

$$B = \exp\left(-ik_{1z}a\right)\left[\frac{i}{2}\left(\frac{k_{2z}}{k_{1z}} - \frac{k_{1z}}{k_{2z}}\right)\sin\left(k_{2z}b\right)\right]$$

$$C = \exp\left(ik_{1z}a\right)\left[-\frac{i}{2}\left(\frac{k_{2z}}{k_{1z}} - \frac{k_{1z}}{k_{2z}}\right)\sin\left(k_{2z}b\right)\right]$$

$$D = \exp\left(-ik_{1z}a\right)\left[\cos\left(k_{2z}b\right) - \frac{i}{2}\left(\frac{k_{2z}}{k_{1z}} + \frac{k_{1z}}{k_{2z}}\right)\sin\left(k_{2z}b\right)\right]$$

Reflection coefficient for an *N*-period multilayer is

$$\left|r_N\right|^2 = \frac{\left|C\right|^2}{\left|C\right|^2 + \left(\dfrac{\sin gd}{\sin Ngd}\right)^2}$$

Transfer matrix and reflection/transmission spectra

Relation between plane-wave amplitudes

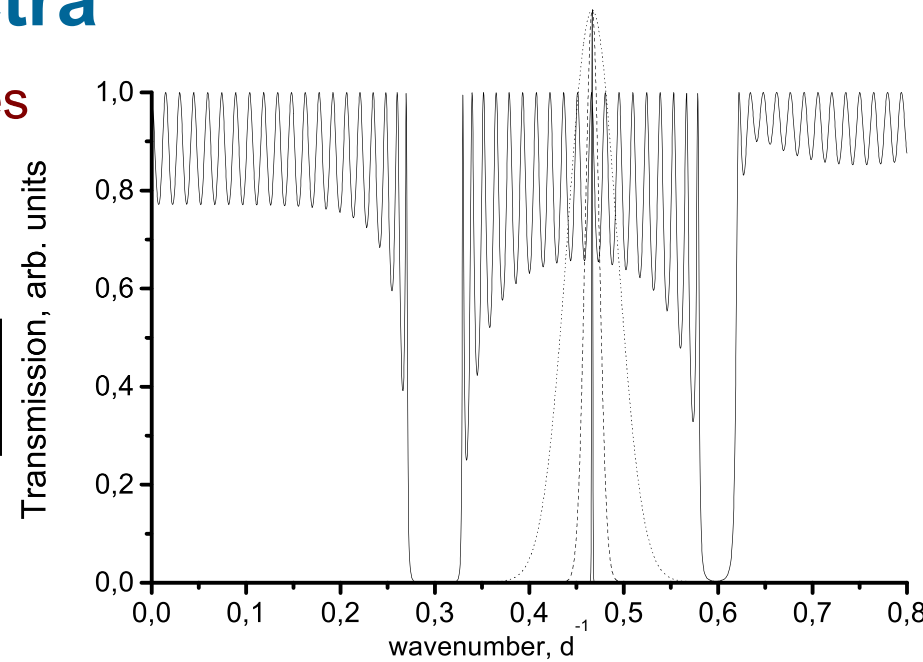
$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

$$A = \exp(ik_{1z}a)\left[\cos(k_{2z}b) + \frac{i}{2}\left(\frac{k_{2z}}{k_{1z}} + \frac{k_{1z}}{k_{2z}}\right)\sin(k_{2z}b)\right]$$

$$B = \exp(-ik_{1z}a)\left[\frac{i}{2}\left(\frac{k_{2z}}{k_{1z}} - \frac{k_{1z}}{k_{2z}}\right)\sin(k_{2z}b)\right]$$

$$C = \exp(ik_{1z}a)\left[-\frac{i}{2}\left(\frac{k_{2z}}{k_{1z}} - \frac{k_{1z}}{k_{2z}}\right)\sin(k_{2z}b)\right]$$

$$D = \exp(-ik_{1z}a)\left[\cos(k_{2z}b) - \frac{i}{2}\left(\frac{k_{2z}}{k_{1z}} + \frac{k_{1z}}{k_{2z}}\right)\sin(k_{2z}b)\right]$$



Transmission of a 20-period multilayer with n_a=1.5, n_b=2 versus spectra of light pulses with τ = 100T_0, 12T_0, 2T_0

Reflection coefficient for an *N*-period multilayer is

$$|r_N|^2 = \frac{|C|^2}{|C|^2 + \left(\frac{\sin gd}{\sin Ngd}\right)^2}$$

Взаимодействие встречных волн

$$\frac{dA}{dz} = -i\kappa B e^{i\Delta\beta z}$$

$$\frac{dB}{dz} = i\kappa^* A e^{-i\Delta\beta z}$$

$$\Delta\beta = \underbrace{\beta_k}_{\text{прямая волна}} - \left(-\beta_k\right) - \frac{2\pi l}{\underbrace{\Lambda}_{\text{вектор обратной решетки}}}$$

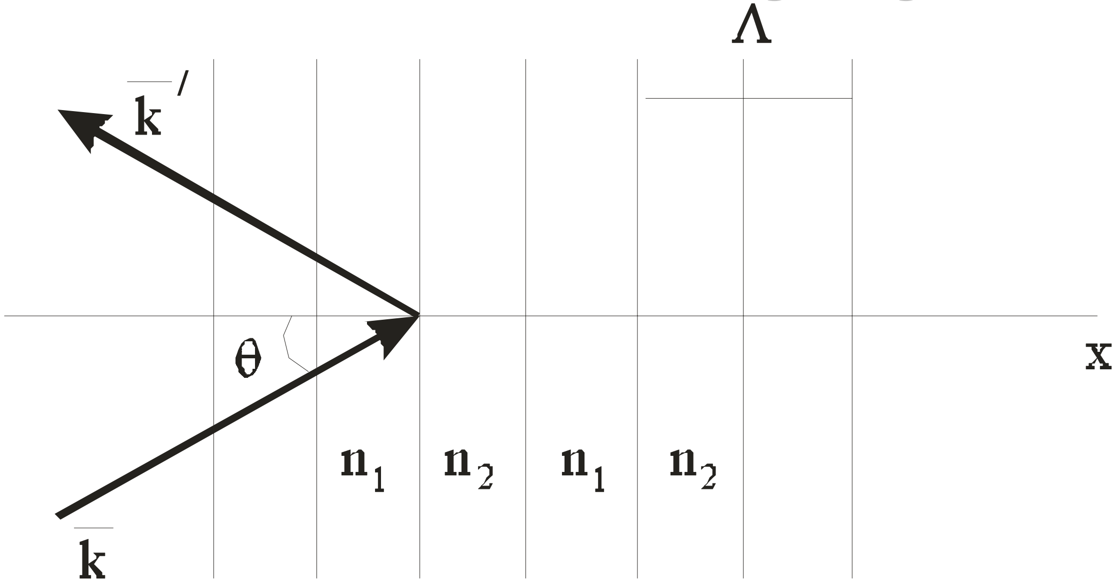
Встречная волна возникает при отражении на скачке показателя преломления

$\Delta\beta = 0$ - фазовый синхронизм (условие Брэгга)

$$\kappa = \frac{\omega}{4}\int \varepsilon_l\left(x\right)\left|A\left(x\right)\right|^2 dx$$

Теория связанных мод для периодической многослойной структуры

$$\varepsilon(x) = \begin{cases} n_2^2, 0 < x < \frac{1}{2}\Lambda, \\ n_1^2, \frac{1}{2}\Lambda < x < \Lambda. \end{cases}$$



$$\varepsilon(x) = \overbrace{\frac{1}{2}\left(n_1^2 + n_2^2\right)}^{\varepsilon_0} + \overbrace{\frac{1}{2}\left(n_2^2 - n_1^2\right)}^{\Delta\varepsilon} f(x)$$

$$f(x) = \begin{cases} 1, 0 < x < \frac{1}{2}\Lambda, \\ -1, \frac{1}{2}\Lambda < x < \Lambda. \end{cases}$$

Коэффициент связи и ширина фотонной запрещенной зоны

$$f(x)=\sum_{m\neq 0}\frac{i(1-\cos\pi m)}{\pi m}\exp\left[-i\frac{2\pi m x}{\Lambda}\right]$$

Коэффициент Фурье

$$|\kappa|=\frac{\omega c}{4\pi m}(1-\cos\pi m)\frac{n_2^2-n_1^2}{\sqrt{n_1^2+n_2^2}}\sqrt{2}$$

$$|\varepsilon_m|=\frac{1}{2}\left(n_2^2-n_1^2\right)\frac{|1-\cos\pi m|}{\pi m}$$

$$\Delta\beta_g=\frac{\omega c}{\pi m}(1-\cos\pi m)\sqrt{2}\frac{n_2^2-n_1^2}{\sqrt{n_1^2+n_2^2}}$$

$$\Delta\beta=2\frac{\pi}{c}\Delta\omega\cos\theta$$

$$\left(\frac{\Delta\omega}{\omega}\right)_g=\frac{c^2}{\pi m\cos\theta}\frac{1}{\sqrt{2}}(1-\cos\pi m)\frac{n_2^2-n_1^2}{n_1^2+n_2^2}$$

$$\left(\frac{\Delta\omega}{\omega}\right)_g\sim\frac{\Delta n}{\bar{n}}$$

Коэффициент отражения

$$A(0) = A_0 \qquad B(L) = 0 \qquad s^2 = \kappa\kappa^* - \left(\frac{\Delta\beta}{2}\right)^2$$

$$R = \left|\frac{B(0)}{A(0)}\right|^2 = \frac{\kappa\kappa^* \operatorname{sh}^2 sL}{s^2 \operatorname{ch}^2 sL + \left(\frac{\Delta\beta}{2}\right)^2 \operatorname{sh}^2 sL}$$

$$\Delta\beta = 2k\cos\theta - \frac{2\pi m}{\Lambda}$$

$$\Delta\beta = 2\frac{\overline{n}}{c}(\omega - \omega_0)\cos\theta \qquad \overline{n} = \frac{\left(n_1^2 + n_2^2\right)^{1/2}}{\sqrt{2}}$$

эффективный показатель преломления

ω_0 - центр фотонной запрещенной зоны

$$\Delta\beta = 0$$

Ширина фотонной запрещенной зоны

$$R_{\max} = \operatorname{th}^2 |\kappa| L$$

$$\Delta\beta_g = 4|\kappa|$$